

## CONCURSUL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”

## ETAPA LOCALĂ

8 februarie 2020

BAREM

CLASA A IX-A

(4 ore/săptămână)

1.)	Din oficiu	1p
	Presupunem $\sqrt{2020^n - 2021} = a \in \mathbb{N}$	1p
	$2020^n - 2021 = a^2 \Rightarrow a = 2k + 1$ (impar), $k \in \mathbb{N}$	2p
	$(2^2 \cdot 5 \cdot 101)^n - 2021 = 4k^2 + 4k + 1 \Rightarrow 2^{2n} \cdot 5^n \cdot 101^n = 4k^2 + 4k + 2022$	3p
	$\underbrace{2^{2n-1} \cdot 5^n \cdot 101^n}_{par} = \underbrace{2k^2 + 2k + 1011}_{impar}$ fals deci $\sqrt{2020^n - 2021} \notin \mathbb{N}$	3p
	Metoda 2) $2020^n - 2021 = M_4 + 3 \Rightarrow \sqrt{2020^n - 2021} \notin \mathbb{N}$	
2.)	Din oficiu	1p
a)	$\frac{a^5 + b^5}{a^3 b^3} \geq \frac{a+b}{ab} \Rightarrow \frac{a^5 + b^5}{a^2 b^2} \geq a+b \Rightarrow a^5 + b^5 \geq a^3 b^2 + a^2 b^3$	1p
	$a^5 - a^3 b^2 + b^5 - a^2 b^3 \geq 0 \Rightarrow a^3(a^2 - b^2) - b^3(a^2 - b^2) \geq 0$	1p
	$(a^2 - b^2)(a^3 - b^3) \geq 0 \Rightarrow (a-b)^2(a+b)(a^2 + ab + b^2) \geq 0$	2p
b)	$x^{1010} = a, y^{1010} = b, z^{1010} = c \Rightarrow \frac{a^2 + b^2}{c^3} + \frac{b^2 + c^2}{a^3} + \frac{c^2 + a^2}{b^3} \geq 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), \forall a, b, c \in \mathbb{R}_+^*$	1p
	$\frac{a^2 + b^2}{c^3} = \frac{a^2}{c^3} + \frac{b^2}{c^3}$ $\frac{b^2 + c^2}{a^3} = \frac{b^2}{a^3} + \frac{c^2}{a^3}$ $\frac{c^2 + a^2}{b^3} = \frac{c^2}{b^3} + \frac{a^2}{b^3}$ $\frac{a^2 + b^2}{c^3} + \frac{b^2 + c^2}{a^3} + \frac{c^2 + a^2}{b^3} = \overbrace{\left(\frac{a^2}{b^3} + \frac{b^2}{a^3}\right)}^{\geq \frac{1}{a} + \frac{1}{b}} + \overbrace{\left(\frac{a^2}{c^3} + \frac{c^2}{a^3}\right)}^{\geq \frac{1}{a} + \frac{1}{c}} + \overbrace{\left(\frac{b^2}{c^3} + \frac{c^2}{b^3}\right)}^{\geq \frac{1}{b} + \frac{1}{c}} \stackrel{i)}{\geq} 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$	4p
3.)	Din oficiu	1p
	$ x + 2020  \in \mathbb{Z} \Rightarrow x \in \mathbb{Z}$	1p
	$\frac{x^2 + x}{2} = \frac{x(x+1)}{2} \in \mathbb{Z}, \forall x \in \mathbb{Z}$	2p

	$ x + 2020  = \frac{x^2 + x}{2} + 2019$	<b>2p</b>
	$x \geq -2020 \Rightarrow x + 2020 = \frac{x^2 + x}{2} + 2019 \Rightarrow x \in \{-1, 2\}$ <b>convin</b>	<b>2p</b>
	$x < -2020 \Rightarrow -x - 2020 = \frac{x^2 + x}{2} + 2019 \Rightarrow x \in \emptyset$	<b>2p</b>

<b>4.)</b>	<b>Din oficiu</b>	<b>1p</b>
	O centrul cercului circumscris și $H_1, H_2, H_3, H_4$ ortocentre <sup>Sylvester</sup> $\Rightarrow$	<b>4p</b>
	$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OH_1}$ (1) $\overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = \overrightarrow{OH_2}$ (2) $\overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} = \overrightarrow{OH_3}$ (3) $\overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OE} = \overrightarrow{OH_4}$ (4)	
	$(1) - (2) \Rightarrow \overrightarrow{OA} - \overrightarrow{OD} = \overrightarrow{OH_1} - \overrightarrow{OH_2} = \overrightarrow{H_2H_1}$ $(4) - (3) \Rightarrow \overrightarrow{OA} - \overrightarrow{OD} = \overrightarrow{OH_4} - \overrightarrow{OH_3} = \overrightarrow{H_3H_4}$	<b>4p</b>
	$\overrightarrow{H_2H_1} = \overrightarrow{H_3H_4} \Rightarrow H_1H_2H_3H_4$ <b>paralelogram</b>	<b>1p</b>